NAME:		( )
CLASS:	TEACHING GROUP:	MARKS /90

PEI HWA SECONDARY SCHOOL

PRELIMINARY YEAR EXAMINATION 2021

Secondary Four (Express)

ADDITIONAL MATHEMATICS **4049/01**

Paper 1

30 August 2021

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Give non-exact numeric answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate. You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

For Examiner's Use	
Category	Question No.
Correction tape	
Pencil written	
Arrows	
Units	
Others	

This question paper consists of <u>19</u> printed pages, inclusive of this cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equations

For the equations $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 Given that A and B are the points of intersection of the line $4y = 2x + 1$ and the curve $\frac{3}{x} - \frac{1}{y} = 4$.

Find the midpoint of AB. [5]

2 The equation of a curve is $y = a + b\sin 3x$, where a and b *are* positive integers.

(a) Given that the maximum and minimum values of y are 6 and -2 respectively, find the value of a and of b. [2]

(b) Explain clearly how the value of a affects the curve. [1]

(c) Sketch the graph of $y = a + b\sin 3x$ for $0° \leq x \leq 180°$. [2]

3 **(a)** Find, in ascending powers of x, the first three terms in the expansion of $\left(1-x+x^2\right)^5$. [2]

(b) Given that the coefficient of x^2 in the expansion of $\left(mx-\frac{1}{x^2}\right)^{11}$, where $m \neq 0$, is 128 times the coefficient of $\frac{1}{x^{10}}$, find the values of m. [4]

4 A function is given by $f(x)=0.05\left[11+(3x+1)^3\right]$ where $x\neq-\frac{1}{3}$.

(a) Determine if f is an increasing or a decreasing function. [2]

The volume of liquid, V m^3, in a container is defined by the function $V=0.05\left[11+(3x+1)^3\right]$ where the height of liquid in the container is x m.

(b) Given that some liquid is poured into the container at a constant rate of 8.1×10^{-2} m^3/s, find the rate of change of x when $V=9.5\times10^{-1}$ m^3. [3]

(c) Briefly explain the meaning of your answer in **(b)**. [1]

5 In a certain firm, there are 50 clients initially.

After 2 years, the number of clients increases to 250.

The number of clients, N, can be modelled by the formula $N = \frac{p}{q + 2^{7-1.5t}}$, where t is the time in years, and p and q are constants.

(a) Find the value of p and of q. [3]

(b) Hence, find the number of years it will take for the firm to have 550 clients. [2]

(c) Explain if the firm will have more than 1000 clients by the end of 10 years. [1]

6 A curve is such that $\frac{d^2y}{dx^2} = 5\cos\frac{1}{6}x + 2\sin\frac{1}{3}x$ and the point $P\left(\pi, -99\sqrt{3}\right)$ lies on the curve.

The gradient of the curve at P is 8.

Find the equation of the curve. [6]

7 The height, h metres, of a ball projected by a canon can be modelled by the equation $h = -0.1d^2 + 4d + 1.8$, where d is the horizontal distance travelled by the ball in metres.

(a) Express $h = -0.1d^2 + 4d + 1.8$ in the form $a(d+b)^2 + c$ where a, b and c are real numbers. [2]

(b) Find the greatest height reached by the ball and the corresponding horizontal distance travelled. [2]

(c) When a ball projected by the canon hits the ground within a horizontal distance of 40 m from the canon, the canon is considered precise.

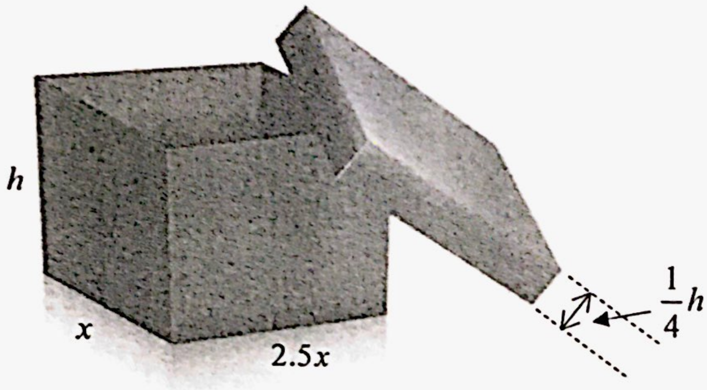
Determine if this canon is precise. [3]

8 Given that $\cos A = \frac{1}{2}$ and $\sin B = -\frac{1}{\sqrt{2}}$ where A and B are in opposite quadrants, find, without using a calculator, the exact value of

(a) $\cos(A - B)$, leaving answer in the form $p\sqrt{2} + q\sqrt{6}$ where p and q are real numbers, [4]

(b) $\cos\frac{A}{2}$. [3]

9 The diagram shows a gift box which consists of a container and its cover. The container has a height of h cm and its length is 2.5 times its breadth, x cm. The height of the cover is $\frac{1}{4}$ times that of the container. The cover fits tightly over the opening of the container.



(a) Given that the volume of the container is 27 000 cm^3, express h in terms of x. [1]

(b) Given that the area of material that is used to laminate the exterior of the container and its cover is A cm^2, show that $A = 5x^2 + \frac{94500}{x}$. [2]

(c) Given that A and x varies, find the value of x for which A has a stationary value and determine whether this value of A is a maximum or a minimum. [4]

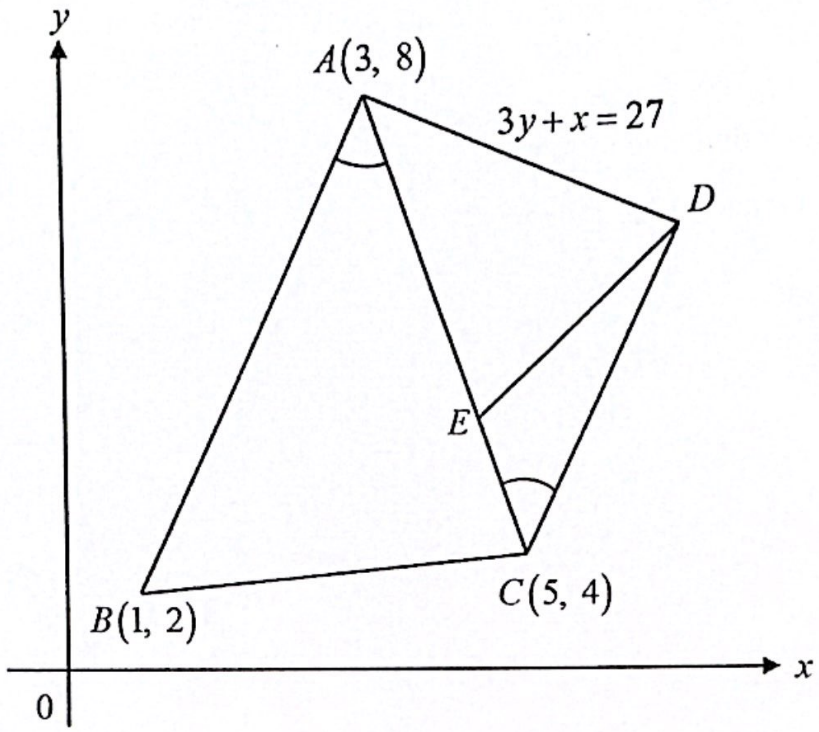
10 The expression $x^3 + ax + b$, where a and b are constants, leaves a remainder of 24 when divided by $x - 3$.

Given that it is exactly divisible by $x + 5$,

(a) find the value of a and of b, [4]

(b) hence, by showing all necessary working, determine the number of real roots of the equation $x^3 + ax + b = 0$. [4]

11



The diagram shows quadrilateral $ABCD$ with vertices $A(3,\ 8)$, $B(1,\ 2)$, $C(5,\ 4)$ and D.

Angle BAC = angle ACD and the side AD has an equation of $3y+x=27$.

(a) Explain why quadrilateral $ABCD$ is a right-angled trapezium. [2]

The point E lies on the line AC such that $AE:EC$ is $3:1$.

(b) Find the coordiantes of E. [2]

(c) Find the area of triangle ADE. [4]

12 (a) Prove the identity $\dfrac{1-\tan x}{\tan x+1}-\dfrac{\tan x+1}{\tan x-1}=2\sec 2x$. [4]

(b) Hence solve the equation $\frac{1-\tan x}{\tan x+1}-\frac{\tan x+1}{\tan x-1}=5$ for $0° < x < 270°$. [3]

(c) Express the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ in terms of π. [1]

13 A particle moves in a straight line so that, t seconds after passing a point O, its velocity, v ms^{-1}, is given by $v = 18 - 6e^{3t}$.

Find

(a) the initial velocity of the particle, [1]

(b) the value of t when the particle comes to an instantaneous rest, [2]

(c) an expression for the acceleration of the particle and state if its velocity is increasing or decreasing, [2]

(d) an expression for the displacement of the particle, [3]

(e) the average speed of the particle in the first 2 seconds of its motion. [3]