Candidate Name: ______________________ (   ) Class: ________

4E
Session 1

KRANJI SECONDARY SCHOOL
Preliminary Examination
Secondary 4 Express

ADDITIONAL MATHEMATICS **4049/02**
Paper 2

Friday **27 August 2021** 2 hr 15 min

Question	Marks
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

READ THESE INSTRUCTIONS FIRST:

Do not open this question paper until you are told to do so.

Write your name, class and register number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

Give non-exact numerical answers correct to three significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

Set by: Ms Priscilla Lee

This Question Booklet consists of 20 printed pages including the cover page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0,$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 **(a)** **(i)** Determine the set of values of *m* for which the equation $2x^2 + 4x + 2m = 6mx - 2$ has real roots. [4]

(ii) Hence, state what can be deduced about the curve $y = 2(x+1)^2$ and the line $y = 6x - 2$. Justify your statement. [2]

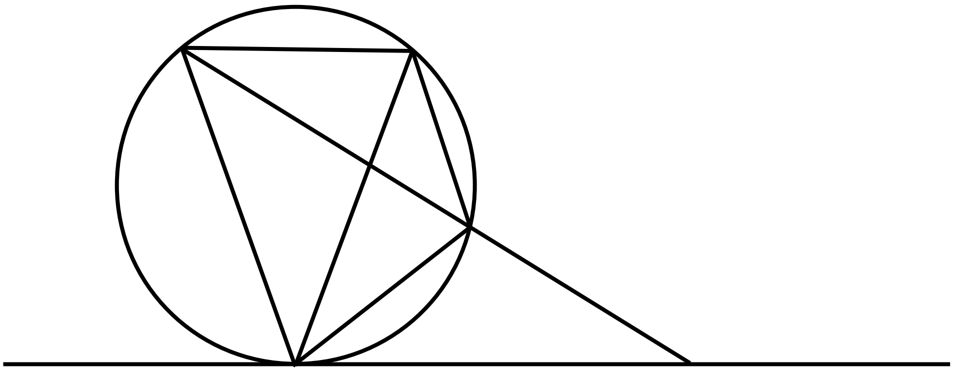
(b) Show that for the curve $y = px^2 + qx + p$ to lie above the line $y = -3(q + px)$, the condition that must apply to the constants p and q is $p < q < 5p$. [4]

2 The coefficient of $\frac{1}{x^3}$ is 512 in the expansion of $\left(\frac{2}{x}+ax^2\right)^9$, where $a < 0$.

(i) Find the value of a. [3]

(ii) Hence, using the value of a found in (i), determine whether the term in $\frac{1}{x^4}$ exists in the expansion $\left(\frac{2}{x}+ax^2\right)^9\left(\frac{1}{8x}+\frac{x^2}{12}\right)$. [3]

3



In the figure above, $CA = CB$ and the line CZ is a tangent to the circle at the point C. AX is produced to meet the tangent at point Y.

(i) Prove that AB is parallel to CY. [3]

(ii) Show that ΔACY and ΔBXC are similar. [3]

4 The table below shows the experimental values of two variables x and y. It is known that one value of y has been recorded incorrectly.

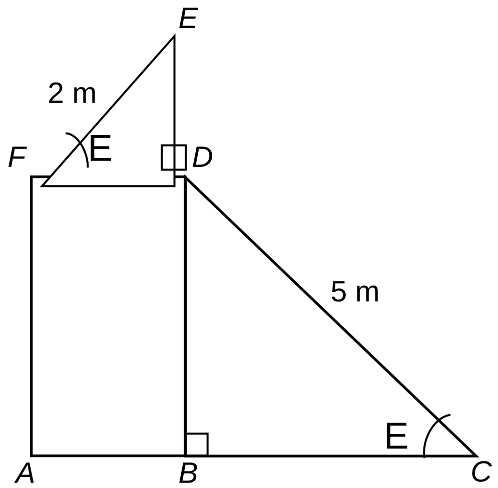
x	0.5	1.0	1.5	2.0	2.5	3.0
y	1.20	1.00	0.86	0.62	0.66	0.59

(i) On the grid on page 9, plot $\frac{1}{y}$ against x, and draw a straight line graph. Use a scale of 2 cm to 0.2 unit on the vertical axis and 2 cm to 0.5 unit on the horizontal axis. [3]

(ii) Find the gradient of your straight line and hence express $\frac{1}{y}$ in terms of x. [2]

(iii) Use your graph to estimate a value of y to replace the incorrect value. [2]

5 It is given that *CD* is 5 m, *FE* is 2 m, $\angle BCD = \angle EFD = \theta$ and the line *BDE* is perpendicular to the line *ABC* as θ varies between 0 and $\frac{\pi}{2}$ radians.

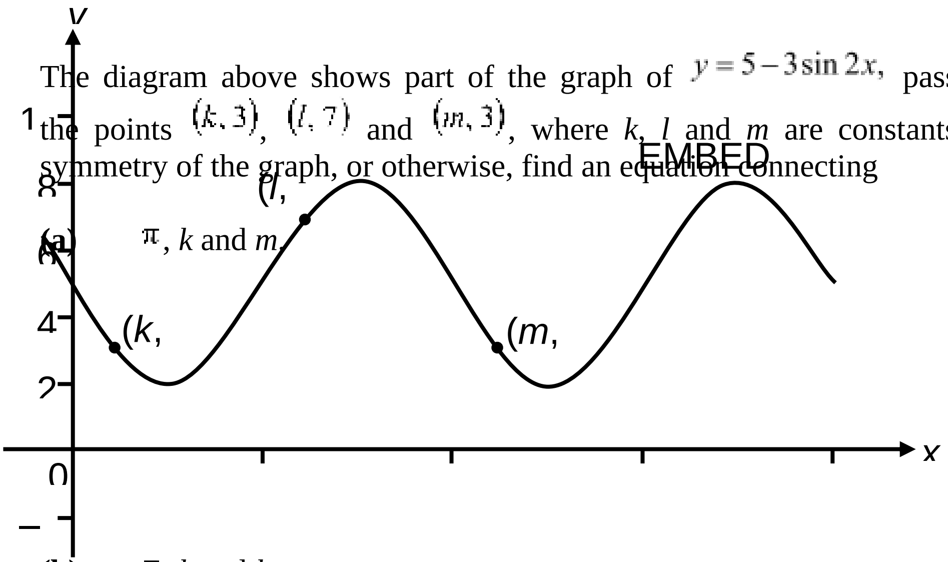


(i) Show that the perimeter *P* of the quadrilateral *ACDF* is *P* = $5+9\cos\theta+5\sin\theta$. [3]

(ii) Express *P* in the form $5+R\cos(\theta-\alpha)$ where *R* is positive and α is acute. Hence find the maximum value of *P* and the corresponding value of θ. [4]

(iii) Does the maximum perimeter of *ACDF* imply that the area of *ACDF* is maximum? Give evidence to support your answer. [4]

6



(i) The diagram above shows part of the graph of $y = 5 - 3\sin 2x,$ passing through the points $(k, 3)$, $(l, 7)$ and $(m, 3)$, where *k*, *l* and *m* are constants. Using the symmetry of the graph, or otherwise, find an equation connecting

(a) π, *k* and *m*. [1]

(b) π, *k* and *l*. [1]

(ii) Explain how the diagram above could be used to determine the number of solution(s) to the equation $1 - 3\sin 2x = \cos\frac{1}{2}x$ for $0 \leq x \leq 2\pi.$ [2]

7 A container with a capacity of 1680 cm^3 is initially fully filled with water. The volume, *V* cm^3, of water in the container is given by $V = h^2 + 2h$ where *h* cm is the depth of water in the container.

(i) Find the initial depth of water in the container. [2]

(ii) Due to a leakage at the bottom of the container, the depth of water decreases at a rate of $\frac{1}{3}t$ cm/s.

(a) Show that $h = -\frac{t^2}{6} + 40$. [3]

(b) Hence, find the rate of change of the volume when t = 6 seconds. [4]

(c) Find the time taken for the container to become empty. [2]

8 **(a)** Evaluate $\log_a 8 \times \log_{16} a.$ [3]

(b) Given that $p = \log_x 2$,

(i) show that $\log_{\frac{4}{x^2}} x^2 = \dfrac{1}{p-1}$. [4]

(ii) Hence solve the equation $\log_x 2 + \log_{\frac{4}{x^2}} x^2 = 3$. [4]

9 **(a)** The value of a new car is $125 800. After m months, the value of the car depreciated by an amount, $$C$, given by $C = 1000\,e^{0.15m}$.

(i) Calculate the amount $C after 1 year, to the nearest dollar. [1]

(ii) Hence, calculate the value of the car after 1 year, to the nearest dollar. [2]

(iii) Calculate the number of months, to the nearest month, when the depreciation exceeded \$28 000. [2]

(b) Without using a calculator, find the value of the integers *a* and *b* for which $\frac{a+\sqrt{b}}{2}$ is the solution of the equation $2x\sqrt{3} + x\sqrt{125} = x\sqrt{45} + \sqrt{12}$. [5]

10 The fuel cost, F, required to operate a machine is known to be proportional to the square of the speed, v km/h.

The total cost of operating the machine includes cost tabulated as below.

Fuel cost (per hour)	\$25 for a speed of 40 km/h
Other costs (per hour)	\$100, regardless of speed

(i) Find the equation connecting the fuel cost per hour, F and the speed of the machine, v. [2]

(ii) Show that the total cost, *C*, in $, in operating the machine for every kilometre is given by the equation $C\text{ (in \$ / km)} = \frac{v}{64} + \frac{100}{v}$. [3]

(iii) Determine the speed that will make the cost per kilometre, *C*, a minimum. [6]

(iv) From **(iii),** find the minimum cost per kilometre. [2]

(v) State one assumption that has been made in the calculations for the above. [1]

End of paper