Core Math CT Notes Y4

Chapter 3 (Quadratic Equations)

$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$ (completing the square)

Quadratic Formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$b^2-4ac > 0$, 2 real soln. $b^2-4ac = 0$, one real soln. $b^2-4ac < 0$, no real soln.

Completed Square Form, $y = a(x-h)^2+k$

Turning point $= (h,k)$

Chapter 4 (Laws of Indices)

$a^0 = 1$ $\quad a^{-p} = \frac{1}{a^p}$ $\quad a^{\frac{1}{p}} = \sqrt[p]{a}$ $\quad a^{\frac{p}{q}} = (\sqrt[q]{a})^p = \sqrt[q]{a^p}$

$a^m \times a^n = a^{m+n}$ $\quad a^m \div a^n = a^{m-n}$ $\quad (a^m)^n = a^{mn}$

$a^n \times b^n = (ab)^n$ $\quad a^n \div b^n = (a \div b)^n$

(Surds)

$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ $\qquad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

$\sqrt{a} \times \sqrt{a} = a$ $\qquad \sqrt{a^2b} = a\sqrt{b}$

To rationalize a fraction

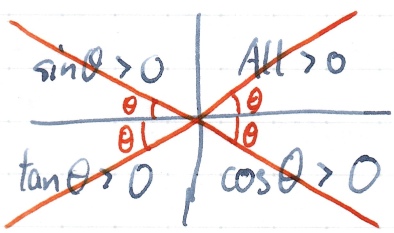
$\frac{8}{\sqrt{3}+\sqrt{7}} \rightarrow \frac{8(\sqrt{3}-\sqrt{7})}{(\sqrt{3}+\sqrt{7})(\sqrt{3}-\sqrt{7})} \rightarrow \frac{8\sqrt{3}-8\sqrt{7}}{3-\sqrt{21}+\sqrt{21}-7} \Rightarrow 2(\sqrt{7}-\sqrt{3})$

(Exponential Eqns.)

If $a^x = a^y$, $x = y$ where $a \neq -1, 0, 1$

Chapter 5 (trig.)

$\sin\theta = \frac{opp}{hyp}$ $\quad \cos\theta = \frac{adj}{hyp}$ $\quad \tan\theta = \frac{opp}{adj}$



Area of $\Delta ABC = \frac{1}{2}ab\sin C$

Sine rule:

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ or $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule:

$a^2 = b^2 + c^2 - 2bc\cos A$

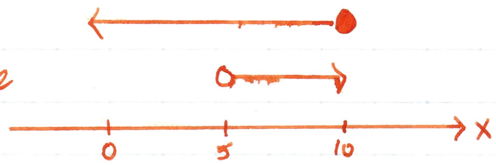
Chapter 6 (Inequalities)

When multiplying or dividing both sides by a negative number, flip the inequality sign.

Number lines

Inclusive

Non-inclusive



Chapter 7 (Sum & Product of Roots)

$\alpha + \beta = -\frac{b}{a} \qquad \alpha\beta = \frac{c}{a}$

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$\alpha^4 + \beta^4 = [(\alpha+\beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2$

To find equation with new roots,

$y = x^2 - (\alpha+\beta)x + (\alpha\beta)$ or $x^2 - (\alpha+\beta)x + \alpha\beta = 0$

(Discriminant)

Discriminant $(D) = b^2 - 4ac$

$D > 0 \Rightarrow$ 2 distinct real roots

$D = 0 \Rightarrow$ 2 repeated real roots

$D < 0 \Rightarrow$ 0 real roots

Chapter 8 (Coordinate Geometry)

Midpoint of AB $= \left(\frac{A_x + B_x}{2}, \frac{A_y + B_y}{2} \right)$

Distance of AB $= \sqrt{(A_x - B_x)^2 + (A_y - B_y)^2}$

Gradient of AB: $\frac{B_y - A_y}{B_x - A_x}$

$l_1 // l_2 \Rightarrow m_1 = m_2$

$l_1 \perp l_2 \Rightarrow m_1 m_2 = -1$

$\hookrightarrow m_1 = -\frac{1}{m_2}$

Shoelace formula

$$\text{Area} = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & \cdots & x_n & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_5 & \cdots & y_n & y_1 \end{vmatrix}$$

Blue arrow means to add the product,
Orange arrow means to subtract the product.

Eqns. of Straight Lines

Given a point (x_1, y_1) and gradient m,

$$y - y_1 = m(x - x_1)$$

OR $y = m(x - x_1) + y_1$

Chapter 10 (Mensuration)

Volume of Pyramid $= \frac{1}{3} \times$ Base Area $\times$ Height

Hence, Vol. of Cone $= \frac{1}{3}\pi r^2 h$

Curved S.A. of Cone $= \pi r l$ where l is the slant height.

$\therefore$ Total S.A of Cone $= \pi r^2 + \pi r l$

Vol. of sphere $= \frac{4}{3}\pi r^3$

S.A. of sphere $= 4\pi r^2$

Chapter 2 (Set Theory)

$A = \{1,2,3,4,5\}$

3 is a member of A and is denoted as $3 \in A$

7 is not a member of A and is denoted as $7 \notin A$

R is the set of real numbers

R^+ is the set of positive real numbers

N is the set of natural/counting numbers i.e. $\{1,2,3,4,5,6,7,\ldots\}$

Z is the set of integers i.e. $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$

Z^+ is the set of positive integers

Empty/null set, $P = \{\}$ or $P = \varnothing$

Sets with no elements in common are called disjoint sets

Every element of P is an element of Q

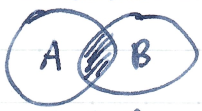
↳ P is a subset of Q $(P \subseteq Q) \Rightarrow n(P) \leq n(Q)$

A subset which is not the same as the original set

↳ Proper subset $(P \subset Q) \Rightarrow n(P) < n(Q)$

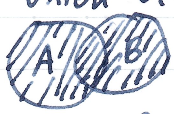
The complement of set B is set B' which contains all elements in the universal set except for elements in set B.

Intersection of sets



$A \cap B$

Union of sets



$A \cup B$

Chapter 3 (Congruency and Similarity)

If two figures are said to be congruent, they are 'identical' to each other in both shape and size.

Tests for congruence in triangles ($\Delta ABC \equiv \Delta DEF$)

- SSS (side-side-side) test
- SAS (side-angle-side) test
  - Angle must be the included angle
- AAS (angle-angle-side) test
- RHS (right angle - hypotenuse - side) test

Similarity

- Same shape, different size

Two triangles are similar if they satisfy one of these properties:

- All the corresponding angles are equal
- All the corresponding sides are proportional
- 2 of the corresponding sides are proportional and the included angles are equal

If figure 1 and figure 2 are similar,

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 \quad \text{and} \quad \frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$

Chapter 4 (Circle Theorem)

Important theorems:

- (tangent ⊥ radius)
- (symm. prop. of tangent)
- (∠ at centre = 2∠ at circumference)
- (∠ in the same segment)
- (⊥ △ in semicircle)
- (alt. seg. thm.)
- (cyclic quad.)

Chapter 5 (Arc Length, Area of Sector and Radian Measure)

Arc length = θr, where θ is in radians

Area of sector = $\frac{1}{2} r^2 \theta$ where θ is in radians